Impact of topology on the dynamical organization of cooperation in the prisoner's dilemma game

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The way cooperation organizes dynamically strongly depends on the topology of the underlying interaction network. We study this dependence using heterogeneous scale-free networks with different levels of (a) degree-degree correlations and (b) enhanced clustering, where the number of neighbors of connected nodes are correlated and the number of closed triangles are enhanced, respectively. Using these networks, we analyze a finite population analog of the evolutionary replicator dynamics of the prisoner's dilemma, the latter being a two-player game with two strategies, defection and cooperation, whose payoff matrix favors defection. Both topological features significantly change the dynamics with respect to the one observed for fully randomized scale-free networks and can strongly facilitate cooperation even for a large temptation to defect, and should hence be considered as important factors in the evolution and sustainment of cooperation.

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I. INTRODUCTION

How cooperation between unrelated individuals emerges and survives in a population when selfish actions are rewarded with a higher benefit is a fascinating area of research. This question has been addressed by studying the evolutionary replicator dynamics [1–3] of simple two-player games such as the prisoner's dilemma, in which each individual can adopt two strategies, cooperation and defection. Both individuals receive R under mutual cooperation and P under mutual defection, while a cooperator receives S when playing with a defector, which in turn receives T, with $T>R>P>S$. Under these conditions, it is obviously better to defect, independent of the opponents strategy. When the strategies are allowed to spread within a population according to the received payoffs, the cooperator density vanishes in the long-time limit if every individual interacts with all other individuals.

Abandoning this "mean-field scenario" and restricting the interaction between individuals according to the topology of a network, where nodes are individuals and edges indicate interactions, it has been shown that cooperation can survive asymptotically [4–15]. The most surprising result is that cooperation can even dominate over defection in heterogeneous scale-free networks, where the number of neighbors of a node (its degree) is broadly distributed. This somewhat counterintuitive behavior has been analyzed in detail very recently [16] by comparing the dynamical organization of cooperation on so-called Barabási-Albert networks [which are scale-free networks with a power-law degree distribution $P(k)\propto k^{-3}$ [17]] and Erdös-Rényi networks (which show a Poisson degree distribution), elucidating the strong impact the degree distribution has on the dynamics.

In this paper, we study how cooperation organizes dynamically in the prisoner's dilemma game when played on heterogeneous scale-free networks with different levels of (a) degree-degree correlations and (b) enhanced clustering, where the number of neighbors of connected nodes are correlated and the number of closed triangles are enhanced, respectively. Besides the degree distribution, these two topological features are the two most characteristic ones and significantly change the dynamics with respect to the one observed for fully randomized scale-free networks, as we show below. This is of importance since most empirically observed social networks display degree-degree correlations and enhanced clustering [18].

II. MODEL

We have created networks of $N=4000$ nodes with a power-law degree distribution $P(k)\propto k^{-\gamma}$ with scale parameter $\gamma=3$ and (a) degree-degree correlations and (b) enhanced clustering. In case (a), the degree-degree correlations are characterized by a mean nearest-neighbor degree $k_{\mathrm{nn}}(k)\equiv\Sigma_j jP(j|k)$ with $P(j|k)$ being the conditional probability that a connection starting at a node of degree k ends at a node of degree j, which roots on the probability $P(j,k)$ that a randomly chosen edge has nodes with degrees j and k at its end as $P(j|k)\equiv P(j,k)/\Sigma_l P(l,k)$. If $k_{\mathrm{nn}}(k)\neq\mathrm{const}$, there are degree-degree correlations present in the network, so that the number of neighbors of connected nodes are correlated. The mean nearest-neighbor degree $k_{\mathrm{nn}}(k)$ is chosen to follow a functional form $k_{\mathrm{nn}}(k)\propto\exp\{(\ln[1+k/k_{\min}])^{\alpha}\}$ with $\alpha\in\{-0.4,-0.2,0,0.2\}$. The networks are created using the algorithm discussed in Ref. [19] which is capable of creating networks with given degree-degree correlations and degree distribution. With a minimal and maximal degree chosen as $k_{\min}=2$ and $k_{\max}=40$, $\alpha\in\{-0.4,-0.2\}$ results in so-called disassortative networks with Newman correlation coefficient $r\equiv\sigma_k^{-2}\Sigma_{jk}jk[P(j,k)-jkP(k)P(j)/\langle k\rangle^2]$ [20,21], with $\sigma_k^2\equiv\langle k^3\rangle/\langle k\rangle-\langle k^2\rangle^2/\langle k\rangle^2$ and $\langle k^q\rangle\equiv\Sigma_k k^q P(k)$, of $r=-0.104$ and -0.058, respectively, $\alpha=0.2$ results in assortative networks with $r=0.066$, and $\alpha=0$ in uncorrelated networks. The mean nearest-neighbor degree $k_{\mathrm{nn}}(k)$ as measured in the created networks is shown in Fig. 1(a) and follows well the desired functional form (note that $\langle k_{\mathrm{nn}}\rangle=\langle k^2\rangle/\langle k\rangle$ for uncorrelated networks [19]). In case (b), the effective clustering is characterized by $\tilde{c}(k)\equiv c(k)/\lambda(k)$, where $c(k)=\Sigma_{i\in\Upsilon(k)}c_i/[NP(k)]$ is the degree-dependent clustering and $\lambda(k)=1-(k-1)^{-1}\Sigma_{j=1}^{k}(k-j)P(j|k)$ the corresponding upper limit [22]. Here, $\Upsilon(k)$ is the set of nodes with degree k, and $c_i=2T_i/[k_i(k_i-1)]$ is the clustering of node i

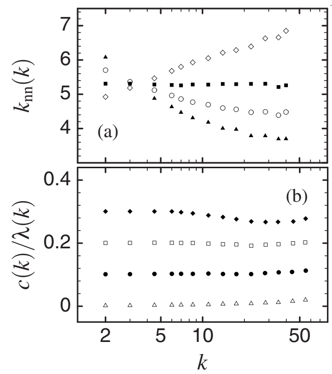

FIG. 1. (a) Measured mean nearest-neighbor degree $k_{\text{nn}}(k)$ vs degree k, for $\alpha=-0.4$ (closed triangle), -0.2 (open circle), 0 (closed square), and 0.2 (open diamond). As $\langle k\rangle=3.08$ and $\langle k^2\rangle=16.1$ for $\alpha=0$, one has $\langle k_{\text{nn}}\rangle=5.23$. (b) Measured effective clustering $c(k)/\lambda(k)$ vs degree k for $\tilde{c}=0$ (open triangle), 0.1 (closed circle), 0.2 (open square), and 0.3 (closed diamond). Here, $\langle k\rangle=3.12$ and $\langle k^2\rangle=18.2$, so that $\langle k_{\text{nn}}\rangle=5.83$.

with T_i being the number of triangles it is part of. We choose $\tilde{c}(k)=\tilde{c}$ with $\tilde{c}\in\{0,0.1,0.2,0.3\}$. The networks are created using a variant [23] of the algorithm discussed in Ref. [22] which is capable of adjusting simultaneously the degree-degree correlations (which we set to zero here). With a minimal and maximal degree chosen as $k_{\text{min}}=2$ and $k_{\text{max}}=60$, $\tilde{c}=0$ results in networks without enhanced clustering, whereas $\tilde{c}>0$ yields networks with enhanced clustering, i.e., enhanced number of closed triangles. The effective clustering $c(k)/\lambda(k)$ as measured in the created networks is shown in Fig. 1(b). Due to topological constraints, the observed effective clustering $c(k)/\lambda(k)$ inevitably drops slightly below the desired value for very large values of $\tilde{c}$, see Refs. [22,23], but otherwise the desired clustering is well-achieved. Note that $\alpha=0$ in case (a) and $\tilde{c}=0$ in case (b) both refer to an ensemble of networks without degree-degree correlations and without enhanced clustering, but with different values of the maximal degree k_{max} [24].

On the largest component of each created network (which usually contains all 4000 nodes up to single ones), the dynamics is implemented similarly as in Ref. [16]: (i) At the beginning, each individual i of the population (i.e., each node i) has the same probability of choosing cooperation or defection as the initial strategy. (ii) Following Refs. [4,9,10,16], we choose the prisoner's dilemma payoffs as $R=1$, $P=S=0$, and $b\equiv T>1$, so that the temptation to defect b is the only parameter, and implement one possible finite population analog of the replicator dynamics [9,10]: At each time step t, which represents one generation of the discrete evolutionary time, each node i in the network plays with all its k_i neighbors and accumulates its obtained payoff π_i. Then, all individuals i synchronously update their strategies s_i by each one choosing one of its neighbors at random, say j, and comparing their respective payoffs π_i and π_j. If the neighbor's payoff is lower or equal, $\pi_j\leq\pi_i$, individual i keeps its strategy s_i for the next time step. On the contrary, if the neighbor's payoff is higher, $\pi_j>\pi_i$, i adopts the strategy s_j of j for the next time step with probability $P(s_i\rightarrow s_j)=(\pi_j-\pi_i)/(b\max\{k_i,k_j\})$ [9,10]. We let the dynamics run for a transient time of 5000 generations. Then, the cooperator density is measured, the dynamics is evolved for another 1000 time steps, and the cooperator density is measured again. If both densities, each averaged over 10 time steps and separated by 1000 generations, deviate by more than 0.01, the procedure is repeated another 1000 generations later. Otherwise, the actual measurement is started over the next 10 000 time steps. To identify the topological aspects of the distribution of cooperators and defectors, we follow Refs. [15,16] in defining pure cooperators and pure defectors (i.e., fixed strategists) and fluctuating agents. All data presented hereafter is averaged over 100 network realizations with ten independent dynamics with different initial conditions on each network realization, resulting in 1000 dynamics per data point.

III. RESULTS AND DISCUSSION

A. Degree-degree correlations

When looking at how the evolutionary dynamics is affected by the degree-degree correlations, one notices that cooperators have marginally worse chances in assortative than in disassortative networks for small temptation to defect b, but considerably better chances for large b, the latter being already conjectured in previous work [6] (note that the Barabási-Albert networks studied in Ref. [16] are slightly disassortative with a mean Newman correlation coefficient $r=-0.06$ for $N=4000$, cf. Sec. III C). The cooperator density ρ changes significantly as a function of the degree-degree correlations, see Fig. 2(a), and the difference can be up to a factor of 2 for large temptation to defect b, see Fig. 3(a) for the ratio $\rho/\rho^{\alpha=0}$, despite that the range of assortativity-disassortativity covered is rather small. The pure cooperator density ρ_{c} (nodes that cooperate during the whole observation period) changes somewhat [see Figs. 2(b) and 3(b), the deviation of $\rho_{\text{c}}/\rho_{\text{c}}^{\alpha=0}$ from 1 for $b\gtrsim 2$ is mainly due to the very small pure defector density $\rho_{\text{c}}^{\alpha=0}$]. An intermediate behavior is observed in the mean pure defector density ρ_{d} (nodes that defect during the whole observation period), see Figs. 2(c) and 3(c). The pure cooperators are located on a single connected component in the vast majority of the cases, as measured by the mean number C_{c} of such components, see Fig. 2(d), which has already been observed for Barabási-Albert networks [16] (the decrease of C_{c} below 1 for large temptation to defect b is caused by realizations without pure cooperators). The surface-to-volume ratio of this component of pure cooperators, as measured by the mean fraction n_{cf} of pure cooperators which have at least one neighbor with fluctuating strategy, increases with decreasing pure cooperator density ρ_{c} (i.e., increasing temptation to defect b), see Fig. 2(e). The discontinuous behavior of n_{cf} at $b=2$ is caused by stable interactions between pure cooperators and pure defectors which occur for this integer value of b. As can be seen in the mean degree κ of cooperator nodes shown in Fig. 2(f), assortativity of a network causes the cooperators to be located on the nodes with a larger degree. This can be understood by looking at the correlations between the degree k_i of a node i and its state ρ_i. Nodes with a large degree tend to

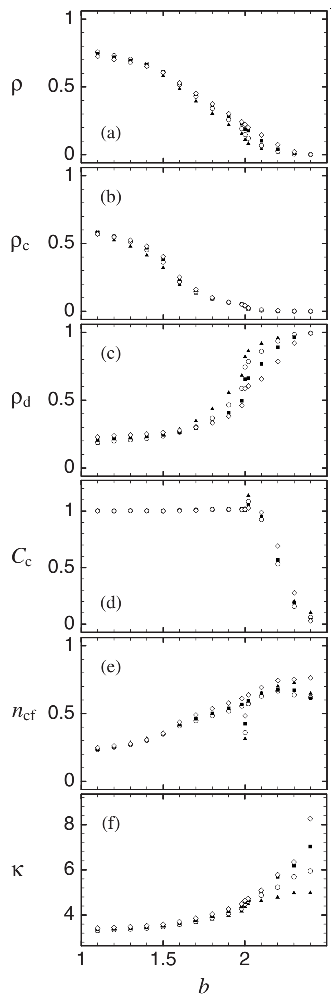

FIG. 2. (a) Mean cooperator density ρ, (b) mean pure cooperator density ρ_c, (c) mean pure defector density ρ_d, (d) mean number of components C_c of pure cooperators, (e) mean fraction n_{cf} of pure cooperators which have at least one neighbor with fluctuating strategy, and (f) mean degree κ of cooperator nodes vs temptation to defect b for networks with a different amount of degree-degree correlations; symbols are as in Fig. 1(a).

cooperate [indicated by a large positive correlation coefficient between ρ_i and k_i^2 in Fig. 4(a), we plot the correlation coefficient with k_i^2 instead of k_i, as this correlation coefficient is somewhat larger which emphasizes the role of nodes with a large degree]; but even more important than the degree k_i of a node i for its state ρ_i is the mean neighbor degree $k_{i,\text{nn}}$ [indicated by an even larger positive correlation coefficient between ρ_i and $k_{i,\text{nn}}^2$ in Fig. 4(b)]. Thus in assortative networks these two effects combine to lead to a large mean degree of cooperators, whereas in disassortative networks these two effects counteract as nodes with a large degree are most likely connected to nodes with a small degree.

The correlations shown in Fig. 4 can be explained by the observation that solely nodes with a large degree can initially afford to be a cooperator and still convince their neighbors to take over their strategy as they have more interactions in which to accumulate payoff. Small degree neighbors of cooperators on nodes with a large degree are easily convinced

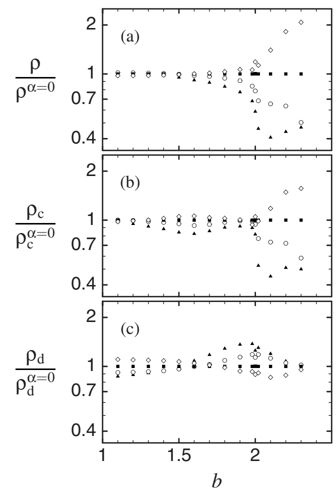

FIG. 3. (a) Mean cooperator density ratio $\rho/\rho^{\alpha=0}$, (b) mean pure cooperator density ratio $\rho_c/\rho_c^{\alpha=0}$, and (c) mean pure defector density ratio $\rho_d/\rho_d^{\alpha=0}$ vs temptation to defect b for networks with a different amount of degree-degree correlations; symbols are as in Fig. 1(a).

to become cooperators as well, and thus an environment with large degrees is positively correlated with a cooperating strategy. Rong *et al.* correctly argued in Ref. [25] that nodes with a large degree connected to each other which share many neighbors will end up as defectors for a large temptation to defect if one of them is initially a defector. From this they concluded that assortativity should therefore be a hindrance to cooperation, taking, however, assortativity to a rather extreme point. Their argument implies that in assortative networks nodes with a large degree tend to share a common neighborhood, which is, however, true only if the degree-degree correlations are so strong that the network disintegrates into mostly homogeneous subgraphs, which ultimately destroys the heterogeneity of the observed networks. This is not the case in heterogeneous complex networks as created by our algorithms, and consequently the shared neighborhood of two nodes with a large degree is rather

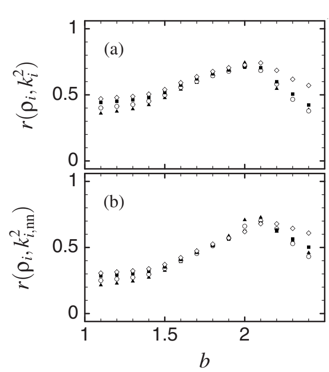

FIG. 4. (a) Mean correlation coefficient $r(\rho_i,k_i^2)$ between the cooperator density ρ_i and the squared degree k_i^2 of site i and (b) mean correlation coefficient $r(\rho_i,k_{i,\text{nn}}^2)$ between the cooperator density ρ_i and the mean squared neighbor degree $k_{i,\text{nn}}^2$ of site i vs temptation to defect b for networks with a different amount of degree-degree correlations; symbols are as in Fig. 1(a).

small, regardless of the degree-degree correlations. What happens for two nodes with a large degree, one being a defector and one being a cooperator, with a distinct neighborhood is that the defector convinces its neighbors with a smaller degree to defect and the cooperator convinces its neighbors with a smaller degree to cooperate. After just a couple of played rounds, the payoff of the cooperator will exceed the payoff of the defector, and there is only a small time window in which the defector might turn the strategy of the cooperator, after which the cooperator will ultimately turn the defector into a cooperator, resulting in two connected cooperator nodes with a large degree. This explains why assortative networks have a higher level of cooperation than disassortative networks for large temptation to defect ($b \approx 2$), as there are more connections between nodes with a large degree. For small temptation to defect ($b \gtrapprox 1$), the situation is different as defectors can only survive in cliques of nodes with a small degree which are more abundant in assortative than in disassortative networks.

B. Enhanced clustering

A similarly large yet different impact on the evolutionary dynamics is observed for different levels of enhanced clustering, i.e., of enhanced number of closed triangles. Networks with enhanced clustering favor the inferior strategy (the strategy having the lower density), which indicates a certain amount of niche building. This can be seen in the cooperator density ρ, the pure cooperator density ρ_c, and the pure defector density ρ_d [see Figs. 5(a)–5(c)], which all show a crossing of the curves for different levels of effective clustering $\tilde{c}$ at approximatively the same density 0.5 and hence for different temptation to defect b. As a consequence, cooperation remains significant for a larger interval of b and the densities ρ, ρ_c, and ρ_d can differ by factors of 3, 10, and 2, respectively, see Figs. 6(a)–6(c). Interestingly, enhanced clustering causes the pure cooperators to get spread over several mutually disconnected components, as indicated by the mean number C_c of such components, see Fig. 5(d). These different components have a small surface-to-volume ratio [as measured by the mean fraction n_{cf} of pure cooperators which have at least one neighbor with fluctuating strategy, see Fig. 5(e)], which indicates that the pure cooperators occupy modules which are well-connected internally but only poorly to the outside, helping cooperation to remain significant for larger temptation to defect b (for enhanced clustering $\tilde{c}>0$, n_{cf} even decreases for $b>2$). The discontinuous behavior of n_{cf} at $b=2$ is again caused by stable interactions between pure cooperators and pure defectors which occur for this integer value of b. Similar to assortativity, enhanced clustering causes an increased cooperator density for large temptation to defect b, but in difference to that case, the mean degree κ of cooperator nodes decreases with increasing clustering, see Fig. 2(f). The reason is that the significant increase in the absolute number of cooperators is the strongest factor determining their mean degree because cooperators tend to be on nodes with the largest degree available in the network, and as the absolute number of cooperators increases with increasing clustering, more nodes of smaller degree become cooperators, leading to a decrease of the mean degree κ of cooperators.

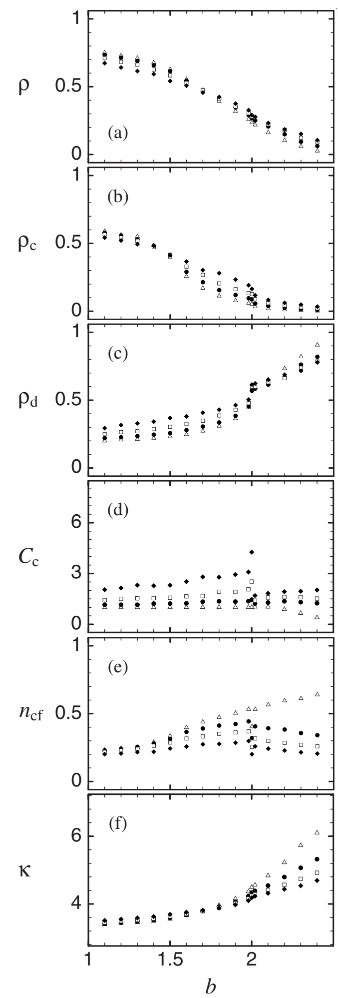

FIG. 5. (a) Mean cooperator density ρ, (b) mean pure cooperator density ρ_c, (c) mean pure defector density ρ_d, (d) mean number of components C_c of pure cooperators, (e) mean fraction n_{cf} of pure cooperators which have at least one neighbor with fluctuating strategy, and (f) mean degree κ of cooperator nodes vs temptation to defect b for networks with a different amount of enhanced clustering; symbols are as in Fig. 1(b).

The present data also allows one to draw some conclusions concerning the influence of the maximal degree k_{max} on the evolutionary dynamics by comparing the case $\alpha=0$ in Fig. 2 with the case $\tilde{c}=0$ in Fig. 5, which differ by the value of the maximal degree k_{max} [24]. One notices that cooperators have better chances in networks with a larger value of maximal degree k_{max}, which was already expected from previous work [16], in particular for large temptation to defect b (note that Barabási-Albert networks studied in Ref. [16] have no fixed value of k_{max} which consequently fluctuates strongly between different network realizations and the mean value of the achieved maximal degree is much higher, $\langle k_{max}\rangle=169.2$ for $N=4000$, so that cooperators have a significant contribution to the dynamics even for $2<b\leq 3$). In our case, the mean degree $\langle k\rangle$ of the network also changes inevitably with k_{max},which might have an influence as well. The differences

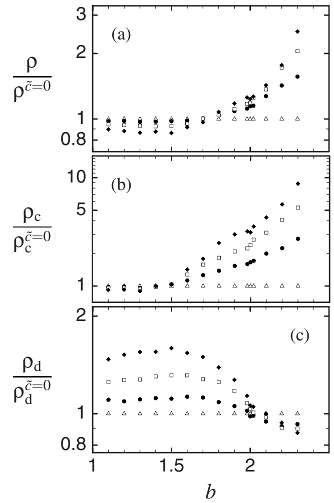

FIG. 6. (a) Mean cooperator density ratio $\rho/\rho^{\tilde{c}=0}$, (b) mean pure cooperator density ratio $\rho_c/\rho_c^{\tilde{c}=0}$, and (c) mean pure defector density ratio $\rho_d/\rho_d^{\tilde{c}=0}$ vs temptation to defect b for networks with a different amount of enhanced clustering; symbols are as in Fig. 1(b).

induced by different values of k_{max} are on the order of magnitude of the differences induced by degree-degree correlations or enhanced clustering.

C. Comparison with reshuffled Barabási-Albert networks

Our results concerning the effect of assortativity are apparently in contradiction to recent studies of the prisoner's dilemma game on Barabási-Albert networks with different direction and probability of reshuffling (i.e., assortative and disassortative mixing) [25], where it has been found that assortativity diminishes cooperation, whereas we observe the opposite effect. To elucide this apparent contradiction, we produced such reshuffled Barabási-Albert networks with $N=4000$ nodes and mean degree $\langle k\rangle=4$, where the direction and probability of reshuffling was chosen such that the resulting Newman correlation coefficients r are similar to the ones we use above. The resulting mean nearest-neighbor degree $k_{nn}(k)$ is shown in Fig. 7. As this function is not monotonously increasing for assortative mixing, these networks are assortative only on average, as measured by the Newman correlation coefficient r. Barabási-Albert networks and especially reshuffled Barabási-Albert networks have certainly their right to being studied, in particular if one assumes some growth mechanism for the network (for instance, in the direction of an evolution of the underlying network with the dynamics [26]). However, the value of $k_{nn}(k)$ is smaller for nodes with large degree k than for nodes with intermediate degree for assortative mixing, hence no clear conclusion concerning the effect of assortativity is possible using these networks.

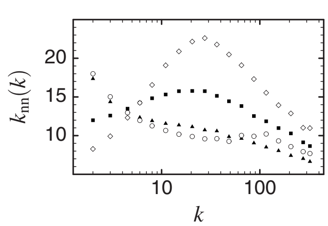

FIG. 7. Measured mean nearest-neighbor degree $k_{nn}(k)$ vs degree k, for Barabási-Albert networks with different direction and probability of reshuffling, disassortative mixing with $p=0.1956$ (closed triangle, Newman correlation coefficient $r=-0.058$), without reshuffling (open circle, $r=-0.055$), assortative mixing with $p=0.1564$ (closed square, $r=0$) and $p=0.4781$ (open diamond, $r=0.067$). As $\langle k\rangle=4$ and $\langle k^2\rangle=54$, one has $\langle k_{nn}\rangle=14$. For Barabási-Albert networks the maximal degree k_{max} is not fixed, and one finds $\langle k_{max}\rangle=169.2$.

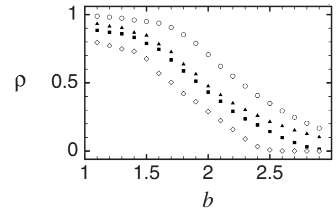

FIG. 8. Mean cooperator density ρ vs temptation to defect b for Barabási-Albert networks with different direction and probability of reshuffling; symbols are as in Fig. 7. Note the slightly larger scale on the abscissa with respect to Figs. 2 and 5.

We have analyzed the evolutionary dynamics of the prisoner's dilemma game on these networks and plot the mean cooperator density ρ in Fig. 8. This data reproduces the results of Ref. [25] regarding assortative and disassortative mixing (the differences are due to the fact that we use $N=4000$ instead of $N=5000$ nodes to allow for a comparison with our data presented above). As Barabási-Albert networks with assortative or disassortative mixing are not simply assortative or disassortative in the sense of a monotonous mean nearest-neighbor function $k_{nn}(k)$, the dynamics is not solely governed by degree-degree correlations. The differences to our networks become particularly clear when looking on the correlations between the state ρ_i and the squared degree k_i^2

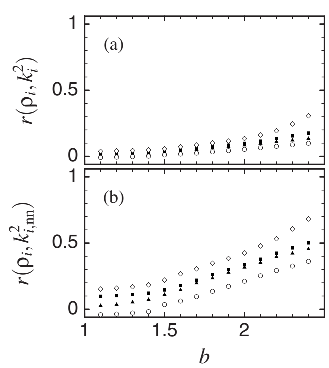

FIG. 9. (a) Mean correlation coefficient $r(\rho_i,k_i^2)$ between the cooperator density ρ_i and the squared degree k_i^2 of site i and (b) mean correlation coefficient $r(\rho_i,k_{i,nn}^2)$ between the cooperator density ρ_i and the mean squared neighbor degree $k_{i,nn}^2$ of site i vs temptation to defect b for Barabási-Albert networks with different direction and probability of reshuffling; symbols are as in Fig. 7.

and the mean squared neighbor degree $k_{i,\mathrm{nn}}^2$ of node i, shown in Fig. 9, which are much smaller when compared to those shown in Fig. 4.

IV. CONCLUSIONS

We have studied how cooperation organizes dynamically in the prisoner's dilemma game when played on heterogeneous scale-free networks with different levels of (a) degree-degree correlations and (b) enhanced clustering, where the number of neighbors of connected nodes are correlated and the number of closed triangles are enhanced, respectively. These two topological features are the two most characteristic ones of a given network besides the degree distribution and significantly change the dynamics with respect to the one observed for fully randomized scale-free networks. Assortative degree-degree correlations substantially enhance cooperation for temptation to defect $b \approx 2$ whereas enhanced clustering considerably reduces the surface-to-volume ratio of emerging cooperating components, allows several of them to exist, and thus facilitates cooperation even for a very high temptation to defect. Most social networks are assortative and clustered [18], and thus it appears reasonable to consider degree-degree correlations and enhanced clustering in interaction networks as important factors in the evolution and sustainment of cooperation.

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